JAMaC 2017

Junior Albemarle Math Competition

Algebra Round

1. Seth is selling bottles of hand sanitizer to elementary school students and the bottles come in two different sizes: Size A and B. Size A costs $3.25 per bottle and size B costs $2.19 per bottle. If Seth sells 3 bottles of size A and makes a total of $9.75, how many bottles of size B did he sell? (2 points)

2. Jesse likes multiplying by 4. He starts with the number 1 and begins multiplying each consecutive number by 4. For example, he could obtain: 1, 4, 16, 64,..etc. If Jesse ends up with 6 numbers, what is the sum of these numbers? (2 points)

3. Ishpreet has a function $f(x) = x^2 - 16$. What is the minimum value of his function? (3 points)

4. Jack throws a ball off of the top of a building. The height of this ball can be modeled by $h(t) = 12t - 5t^2 + 32$, where $h(t)$ is the height of the ball and t is time. At what time does the ball hit the ground? (3 points)

5. A soccer team wants to buy new balls, cleats, and socks. 5 balls, 7 pairs of cleats, and 9 pairs of socks costs $120. 2 balls, 6 pairs of cleats, and 8 pairs of socks costs $96. 2 balls, 3 pairs of cleats, and 4 pairs of socks costs $52. What is the cost of one soccer ball, one pair of cleats, and one pair of socks? (4 points)

6. Max can run a full lap around the AHS track in 1 minute and 43 seconds. Derek can run a full lap around the same track in 1 minute and 26 seconds. If they begin running around the track at the same time and location, how many full laps will Max have run before Derek passes him for the first time? (4 points)

7. Brennan is thinking of three positive integers. The sum of the two largest integers is 77 and the product of the two smallest integers is 494. Find the difference between the largest and smallest integers. (5 points)

8. Let $f(x) = 4x^4 + 6x^3 - 12x^2 + 789x - 901$and a, b, c, d be the roots (not necessarily distinct) of $f(x)$. Find the value of $\frac{a^2bcd+ab^2cd+abc^2d+abcd^2}{ab+ac+ad+bc+bd+cd}$.(6 points)

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General Round

1. What is the sum of all of the distinct positive factors of 24? (2 points)

2. Bob has 100 distinct pairs of gloves. If Bob randomly pulls one glove out at a time from his drawer, what is the minimum number of gloves that he must pull out in order to guarantee that he will have a matching pair. (2 points)

3. Let $a = 7x$ where x is a positive integer. How many values of a occur between 1 and 200? (3 points)

4. Thomas is tasked with assigning serial numbers to all the merchandise in Gander Mountain. Each serial number consists of 4 distinct digits, from 0 to 9 inclusive. In addition, the sum of the digits in the serial number must be greater than 6. How many serial numbers are possible? (3 points)

5. Compute the smallest positive integer that leaves a remainder of 3 when divided by 7, a remainder of 2 when divided by 5, and a remainder of 1 when divided by 17. (4 points)

6. A high school is electing members for its 4-person student council. If the candidates for the council include 2 freshmen, 1 sophomore, 3 juniors, and 1 senior, what is the probability that both of the freshmen are elected to the council? (4 points)

7. What is the units digit of the expression: $2017^{2017^{2017}} - 1$? (5 points)

8. Let a 7-digit telephone number $a_1a_2a_3 - a_4a_5a_6a_7$ be *nice* if the sequence $a_1a_2a_3$ is the same as either of (or possibly both) $a_4a_5a_6$ or $a_5a_6a_7$. If each digit can be any of the ten decimal digits $0, 1, 2, .. 9$, what is the number of different *nice* telephone numbers? (6 points)

Geometry Round

1. Derek is building a fence for his pet ravens. The rectangular fence has a perimeter of 14 feet. If the length of the fence is 4 feet, what is the length of the diagonal of the fence **in inches**? (2 points)

2. Ashwin has a slice of pineapple pizza that happens to be in the shape of a right triangle. If the longest side of the pizza has length $5\sqrt{2}$, and the shortest side has length 5, what is the measure of the angle that is opposite from the shortest side of the pizza slice? Express your answer in degrees. (2 points)

3. Max the Mouse has a piece of cheese in the shape of a rectangular prism with a square base. If the side length of the square base is equivalent to $4\sqrt{2}$ inches, and the prism has a height of 6 inches, what is the length of the spatial diagonal of the prism? (3 points)

4. Three distinct circles with radii lengths of 3, 4, and 5 are mutually tangent. What is the area of the triangle formed by connecting the centers of these three circles? (3 points)

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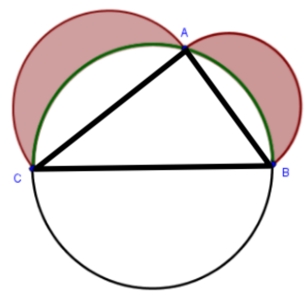
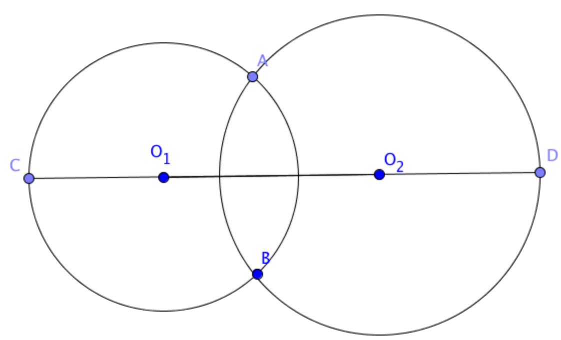
5. Let there be a circle with center O. Given two squares, one inscribed in circle O and one circumscribing O, what is the ratio of the area of the larger square to the area of the smaller square? (4 points)

6. A circle is circumscribed about triangle ABC, which is a right triangle with sides $AB = 3$, $AC = 4$ and $BC = 5$. Semicircles are drawn on sides AB and AC. Find the area between the semicircles and the circle circumscribed about triangle ABC. (4 points)



7. Let O_1 and O_2 be the centers of two distinct circles of radius 4 and 5, respectively that intersect at points A and B with $AB = 6$. If O_1O_2 intersects circles O_1 and O_2 at C and D, respectively, find AD. (5 points)



8. Let $ABCD$ be a quadrilateral such that angles $DAC = 92^o$, $DBC = 88^o$, $BCD = 72^o$.If $AD = BC$, find the angle measure of ACD. (6 points)

Team Round (30 Minutes)

1. If a, b, c, d are real numbers that satisfy the equation $(a - 3)^2 + (4 + b)^2 + (12 - c)^2 + (d + 5)^2 = 0$, compute $ab - cd$. (2 points)

2. Given a cube with an edge of length 10, what is the area of the largest triangle that can be formed by connecting three vertices of the cube? (2 points)

3. Consider the polynomial $4x^4 + 11x^3 + 7x^2 + 6x + 9$. If the roots of this polynomial are $a, b, c,$ and d, compute $\frac{abcd}{a+b+c+d}$. (3 points)

4. Ian wants to make a mixtape. He has a total of 15 songs, 7 of which are “good” and 8 of which are “bad”. He will choose 7 songs to put on his mixtape. Assuming that each song is distinct, how many different mixtapes are possible if the mixtape must have at least 4 “good” songs? (3 points)

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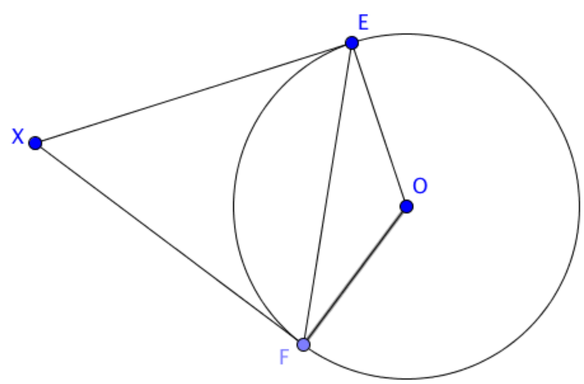
5. What is the units digit of $7^x + 7^{x+3}$ if x has a remainder of 3 when divided by 4? (4 points)

6. What is the probability of getting 4 heads when you flip a coin 8 times if the probability of getting a head is ⅓, while the probability of getting a tail is ⅔? (4 points)

7. Let X be a point the lies outside of a Circle O of radius 3. Lines XF and XE are tangent to O at $E$and F. Given that $XF = 4$, what is the length of EF? (5 points)



8. Given that $x = \sqrt{x + 7\sqrt{x + 7\sqrt{x + 7\sqrt{....}}}}$, find all possible values of x. (5 points)

9. A room is filled with 16 red balloons, 14 blue balloons, and a number of green and purple balloons that are in the ratio 13:7. If 12 purple balloons are added to the room, the probability that when one balloon is selected at random, it isn't purple is 43/62. Find the number of total balloons (after the 12 purple balloons are added). (6 points)

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10. (note: this problem was omitted due to an error.) Let $ABCD$ be a cyclic quadrilateral inscribed in circle O such that angle $BAC = 37^o$ and angle $ACD = 23^o$. Let B' be the intersection of the extension of DB and the line tangent to circle O at point C. Find the angle measure of ACB'. (7 points)

<u>Finalists' Round</u> **(35 Minutes)**

1. How many distinct solutions are there to the equation $x + y + z = 11$, where x, y, and z are positive integers? (4 points)

   A. 19 B. 45 C. 11 D. 44 E. 96

2. Right triangle ABC has a right angle at B and has $AB = x + 2$, $AC = x + 4$. If $BC = 12$, what is the value of x? (4 points)

   A. $13\sqrt{2}$ B. $33\sqrt{2}$ C. 33 D. 32 E. $32\sqrt{2}$

3. What is the side length of the smallest square that contains all points: $(1, 2)$, $(3, -4)$, $(5, 18)$, $(9, 17)$, $(26, -54)$, and $(-15, 8)$? (4 points)
   (Note that points can lie on the edges of the square)

   A. 15 B. 41 C. 45 D. 46 E. 72

4. If x is a non-negative integer, what is the units digit of $9^x + 9^{x+1}$ equivalent to? (3 points)

   Answer: ____________

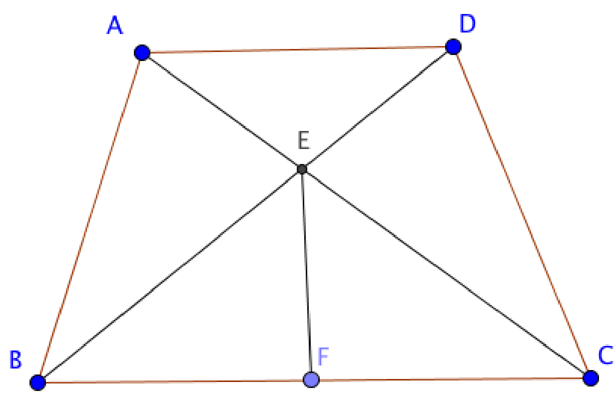
5. In the space below, justify your answer to question #4 in one or two sentences. (3 points)

6. Max is a ladybug on the coordinate plane and is currently located at the origin, or $(0, 0)$. Max's favorite restaurant is Panda Express, which is located at the point $(5, 7)$. If Max can only move either upwards or to the right (one unit at a time), how many different paths are there for Max to travel to Panda Express? (5 points)

   Answer: ____________

7. Let $ABCD$be an isosceles trapezoid of area 48 with AD || $BC$and $AD = 4,\ BC = 8$. AC and BDintersect at point $E$and $F$is the midpoint of BC. If $EF = 4$, find DE. (5 points)

   Answer: ____________



8. Let a, b, c, d be the distinct roots of the polynomial $x^4 + 3x^3 + 4x^2 + 3x + 1$. Compute the number of distinct real roots of the polynomial. (6 points)

   Answer:___________

9. Tim plays a game with a weighted coin that comes up tails with probability $\frac{12}{34}$ and heads with probability $\frac{22}{34}$. If the probability that Tim flips 3 heads after 100 total flips can be expressed as $\frac{a}{b} * (\frac{c}{d})^x * (\frac{e}{f})^y$, compute $a + b + c + d + e + f + x + y$. Note that each fraction is written in simplest form. (6 points)

   Answer: ___________

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10. There exists a quadrilateral $ABCD$ such that angles $A = 120^o$, $B = D = 90^o$. If $AB = 10$ and $AD = 26$, what is the length of AC? (Do not simplify any square roots) (7 points)

Answer:___________